

8. Statistical properties of grayscale images

8.1. Introduction

This laboratory work presents the main statistic features that characterize the distribution of intensity levels in a grayscale image or in an area / region of interest (ROI) of the image. These statistic features can be applied similarly to color images, on each color component.

The following notation will be used throughout this lab:

- $L=255$ maximum intensity level
- $h(g)$ histogram function, counts the number of pixels with gray level g
- $M=H*W$, number of pixels in the image
- $p(g)=h(g)/M$ gray level probability distribution function (PDF).

8.2. The mean value of intensity levels

The mean value of intensity levels is a measure of the mean intensity of the given image or of the region of interest. A dark image has a low mean value (Fig. 8.1a), and a bright image has a high mean value. (Fig. 8.1b).

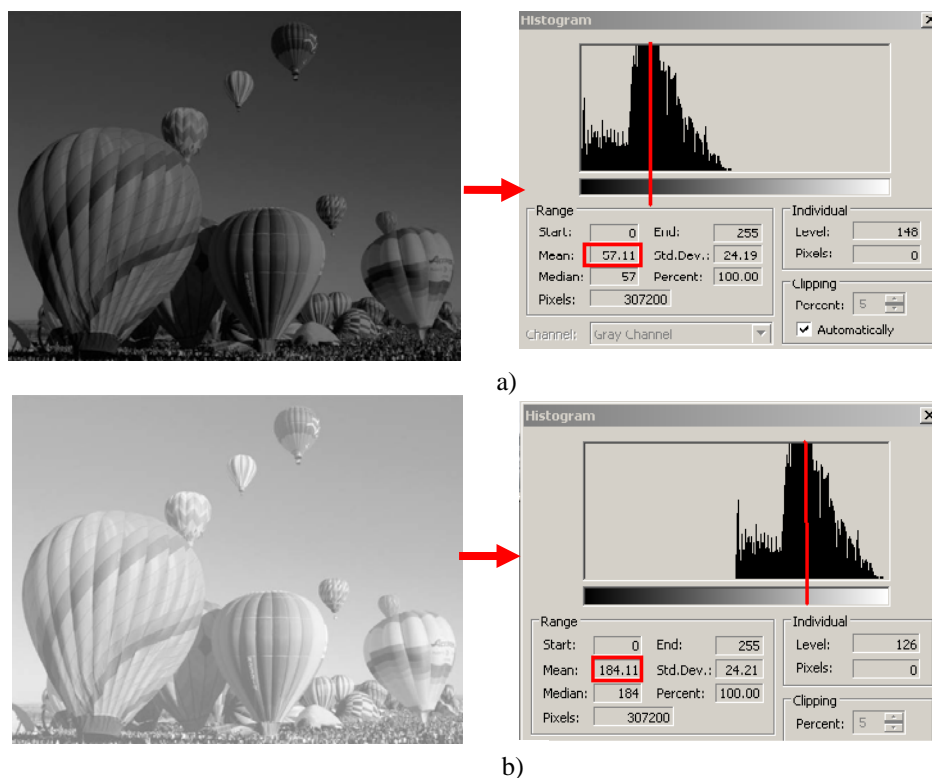


Fig. 8.1 The position of the histogram and the mean value of the intensity levels for a) a dark image and b) for a bright image

The mean intensity value is computed as follows:

$$\bar{g} = \mu = \int g \cdot p(g) dg = \sum_{g=0}^L g \cdot p(g) = \frac{1}{M} \sum_{g=0}^L g \cdot h(g) \quad (8.1)$$

$$\bar{g} = \mu = \frac{1}{M} \sum_{i=0}^{H-1} \sum_{j=0}^{W-1} I(i, j) \quad (8.2)$$

8.3. The standard deviation of the intensity levels

The standard deviation of the intensity levels represents a measure of the contrast of an image (region of interest). It characterizes the dispersion (spreading) of the intensity levels with respect to the mean value. An image having a high contrast will have a large standard deviation (Fig. 8.2a – the histogram is spread on the entire range of intensity levels), and an image having a low contrast will be characterized by a small standard deviation (Fig. 8.2b – the histogram is restricted to some intensity levels located around the mean value).

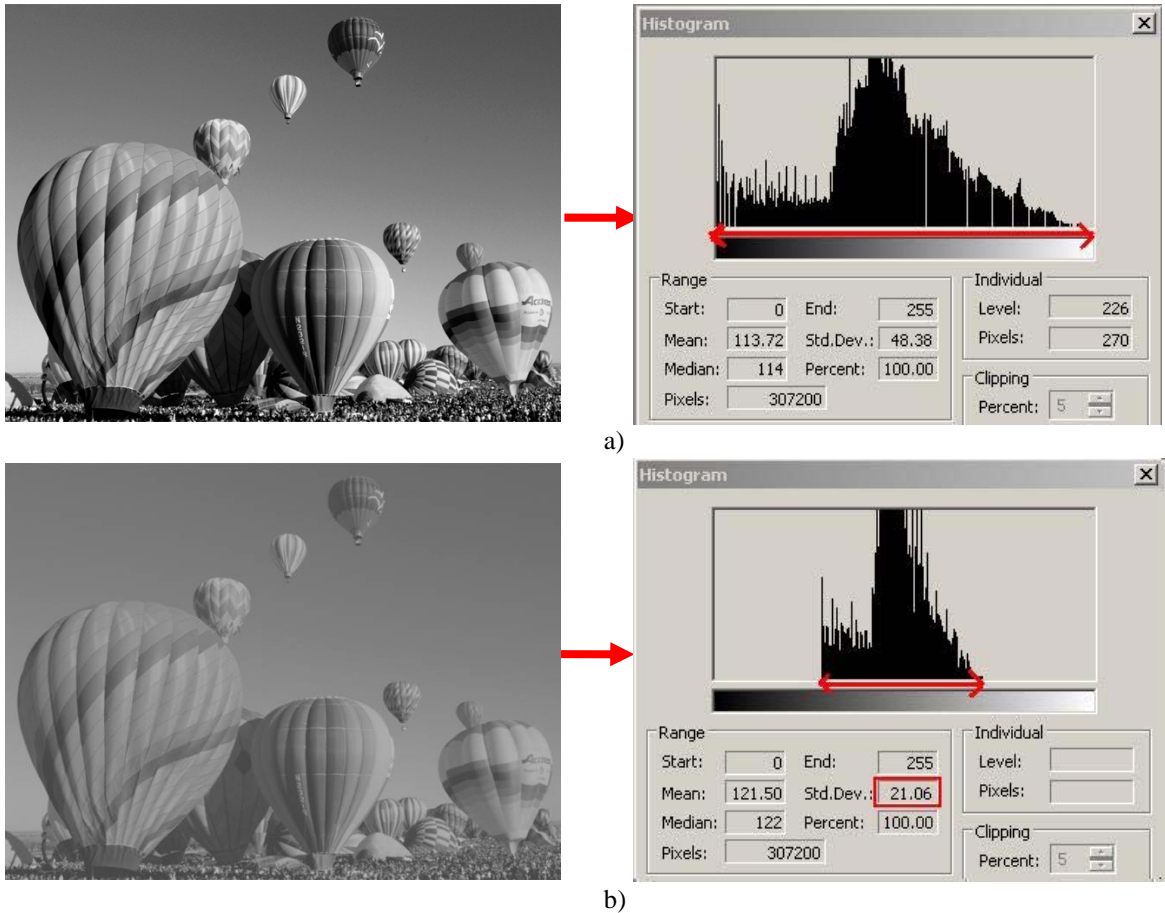


Fig. 8.2 The position of the histogram and of the standard deviation (2σ) of the intensity levels for an image of high contrast (a) and an image of low contrast (b).

The standard deviation of the intensity levels is given by:

$$\sigma = \sqrt{\sum_{g=0}^L (g - \mu)^2 \cdot p(g)} \quad (8.3)$$

$$\sigma = \sqrt{\frac{1}{M} \sum_{i=0}^{H-1} \sum_{j=0}^{W-1} (I(i, j) - \mu)^2} \quad (8.4)$$

8.4. Threshold selection by optimal image approximation

This algorithm determines an optimal threshold, in the sense that the approximation error between the original image and the resulting binary image is minimal. The error can

either be computed as the sum of absolute differences (P1, city block or Manhattan norm) or as the squared sum of differences (the square of a P2 or Euclidean norm). A further improvement is to determine two gray level values for the approximating the image, instead of using black and white.

8.4.1. Minimizing the city block distance

The threshold that minimizes this distance is (see course notes):

$$T = \underset{t}{\operatorname{argmax}} t \sum_{g=t}^L h(g) = \underset{t}{\operatorname{argmax}} tA(t) \quad (8.5)$$

The function A(t) can be computed incrementally by using the recurrence:

$$\begin{aligned} A(t+1) &= A(t) - h(t), \\ A(0) &= \sum_{g=0}^L h(g) = M \end{aligned} \quad (8.6)$$

8.4.2. Minimizing the Euclidean distance

The threshold that minimizes this distance is (see course notes):

$$T = \underset{t}{\operatorname{argmax}} t \sum_{g=t}^L (2g - t)h(g) = \underset{t}{\operatorname{argmax}} tW(t) \quad (8.7)$$

The function W(t) can be computed incrementally by observing that:

$$\begin{aligned} W(t+1) &= \sum_{g=t+1}^L (2g - t - 1)h(g) = \\ &= \sum_{g=t}^L (2g - t - 1)h(g) - (2t - t - 1)h(t) = \\ &= \sum_{g=t}^L (2g - t)h(g) - \sum_{g=t}^L h(g) - (t - 1)h(t) = \\ &= W(t) - A(t) - (t - 1)h(t) \end{aligned} \quad (8.8)$$

where A(t) is computed according to (8.6). We also have that:

$$W(0) = \sum_{g=0}^L 2gh(g) = 2\mu M \quad (8.9)$$

8.4.3. Improving the image approximation by using two non-black&white levels

The optimal threshold T is obtained by maximizing:

$$T = \underset{t}{\operatorname{argmax}} \left(\frac{\left(\sum_{g=0}^{t-1} gh(g) \right)^2}{\sum_{g=0}^{t-1} h(g)} + \frac{\left(\sum_{g=t}^L gh(g) \right)^2}{\sum_{g=t}^L h(g)} \right) = \underset{t}{\operatorname{argmax}} \left(\frac{(\mu M - M_{high}(t))^2}{M - A(t)} + \frac{(M_{high}(t))^2}{A(t)} \right), \quad (8.10)$$

$$t \geq 1 \text{ and } \sum_{g=0}^{t-1} h(g) \neq 0 \text{ and } \sum_{g=t}^L h(g) \neq 0 \quad (M - A(t) \neq 0 \text{ and } A(t) \neq 0)$$

Where A was computed in (8.6), and

$$M_{high}(t+1) = M_{high}(t) - th(t),$$

$$M_{high}(0) = \sum_{g=0}^L gh(g) = \mu M \quad (8.11)$$

Using the determined threshold T, the gray level values that minimize the approximation error are the two medians:

$$g_{low}(T) = \frac{\sum_{g=0}^{g=T-1} gh(g)}{\sum_{g=0}^{g=T-1} h(g)} \quad \text{and} \quad g_{high}(T) = \frac{\sum_{g=T}^{g=L} gh(g)}{\sum_{g=T}^{g=L} h(g)} \quad (8.12)$$

8.5. Histogram analytical transformation functions

In Fig. 8.3 are shown some typical transformation functions of the intensity values, which can be expressed in an analytical form:

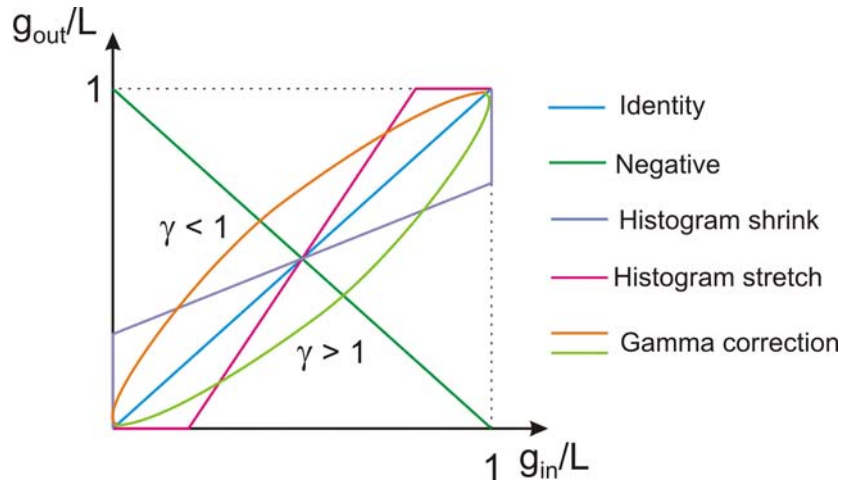


Fig. 8.3 Typical gray levels transformation functions

8.5.1. Identity function (no effect):

$$g_{out} = g_{in} \quad (8.13)$$

8.5.2. Image negative:

$$g_{out} = L - g_{in} = 255 - g_{in} \quad (8.14)$$

8.5.3. Histogram stretching / shrinking:

$$g_{out} = g_{out}^{MIN} + (g_{in} - g_{in}^{MIN}) \frac{g_{out}^{MAX} - g_{out}^{MIN}}{g_{in}^{MAX} - g_{in}^{MIN}} \quad (8.15)$$

Where:

$$\frac{g_{out}^{MAX} - g_{out}^{MIN}}{g_{in}^{MAX} - g_{in}^{MIN}} = \begin{cases} > 1 & \Rightarrow \text{stretch} \\ < 1 & \Rightarrow \text{shrink} \end{cases} \quad (8.16)$$

8.5.4. Gamma correction:

$$g_{out} = L \left(\frac{g_{in}}{L} \right)^\gamma \quad (8.17)$$

Where:

γ is a positive coefficient: < 1 (gamma encoding/compression) or > 1 (gamma decoding / decompression)

Attention: always check that: $0 \leq g_{out} \leq 255$. If outside the domain, values should be saturated!!!

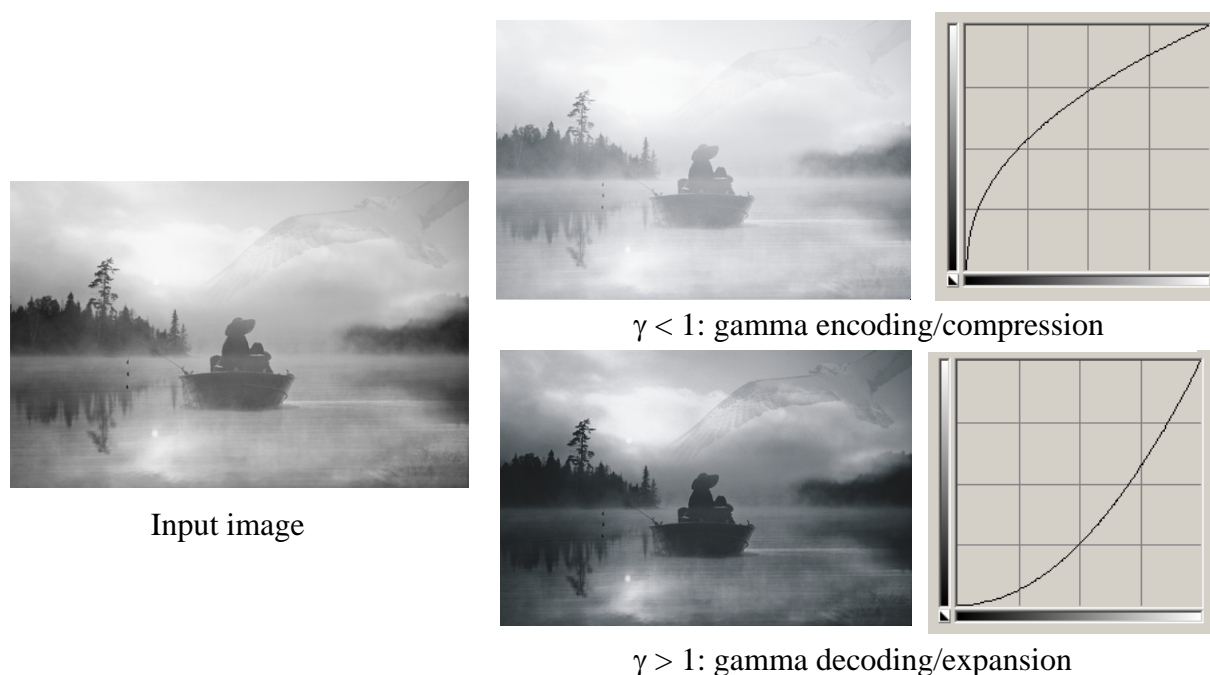


Fig. 8.4 Results of gamma correction operations

8.5.5. Brightness changing (histogram slide)

$$g_{out} = g_{in} + offset \quad (8.18)$$

Attention: always the following checking will be done: $0 \leq g_{out} \leq 255$. If an overflow beyond these limits appears, output values will be truncated or scaled!!!

8.6. Histogram equalization

Histogram equalization is a transform which allows us to obtain an output image with a quasi-uniform histogram/PDF, regardless the shape of the histogram/PDF of the input image. For that purpose, the following transform will be used (see lecture notes for more details):

$$s_k = T(r_k) = \sum_{j=0}^k p_r(r_j) = \sum_{j=0}^k \frac{n_j}{n} \quad , \quad k = 0 \dots L \quad (8.19)$$

Where:

r_k – normalized intensity level of the input image corresponding to the (un-normalized)

intensity level k : $r_k = \frac{k}{L}$, ($0 \leq r_k \leq 1$ and $0 \leq k \leq L$)

s_k – corresponding normalized intensity level of the output image;

$p_C(r_k)$ – cumulative probability density function (CPDF) of the input image

$$p_C(r_k) = \sum_{j=0}^k p_r(r_j) = \sum_{g=0}^k \frac{h_r(g)}{M} \quad (8.20)$$

r_j – normalized intensity level of the input image corresponding to the (un-normalized)

intensity level j : $r_j = \frac{j}{L}$.

8.6.1. Histogram equalization algorithm

1. Compute the histogram or the PDF of the input image (as a 256 elements vector)
2. Compute the CPDF of the input image (8.20), as a vector of 256 elements.
3. Compute the transformation for the histogram equalization according to (8.20).
Because the s_k values obtained from (8.19) are normalized intensity values, it is necessary to transform the normalized intensity values s_k back to un-normalized ones by multiplication with L (the highest intensity value: 255 for 8 bits/pixel images):

$$g_{out} = Ls_k = \frac{L}{M} \sum_{g=0}^{g_{in}} h(g) \quad , \quad k = g_{in} \quad (8.21)$$

This transformation function can be written as an equivalence table (vector):

$$g_{out} = tab(g_{in}) = 255 \cdot p_C(g_{in}) \quad (8.22)$$

4. The intensity values of the output (equalized) image are computed using the equivalence table:

$$lpDst[i * w + j] = tab[lpSrc[i * w + j]] \quad (8.23)$$

8.7. Practical work:

1. Compute and display the mean and standard deviation of image intensity levels.
2. Implement the three functions for automatic threshold computation (section 8.4) and threshold the images according to these values.
3. Implement the histogram transformation functions (section 8.5) for image negative, histogram stretching, gamma correction, histogram slide. Input the limits g_{out}^{MIN} , g_{out}^{MAX} , the gamma coefficient and the brightness increase value from a dialog box.
4. Implement the histogram equalization algorithm (section 8.6).
5. **Save your work. Use the same application in the next laboratories. At the end of the image processing laboratory you should present your own application with the implemented algorithms.**

Bibliography

[1]. R.C.Gonzales, R.E.Woods, *Digital Image Processing. 2-nd Edition*, Prentice Hall, 2002.